Nama : Mohammad Febriyan Khamim
NRP : 5002221072

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Tugas 7 Metode Matematika (D)

Halaman 60 – 61

1). $U_{n+2} - 3U_{n+1} + 2U_n = 0$

Karena $U_{n+2} = E^2 U_n$; $U_{n+1} = E U_n$, maka

$$(E^2 - 3E + 2) U_n = 0$$

Substitusi $U_n = p^n$, maka :

PP : $p^2 - 3p + 2 = 0$

$(p-1)(p-2) = 0$

$p = 1 \vee p = 2$

Jadi, diperoleh : $U : C_1 + C_2 . 2^n$ //

5) $y_{n+2} - 7y_{n+1} + 12y_n = 0$

Karena $y_{n+2} = E^2 y_n$; $y_{n+1} = E y_n$, maka :

$$(E^2 - 7E + 12) y_n = 0$$

Substitusi $y_n = p^n$, maka :

PP : $p^2 - 7p + 12 = 0$

$(p-3)(p-4) = 0$

$p = 3 \vee p = 4$

Jadi, diperoleh : $y = C_1 3^n + C_2 4^n$ //

7). $\Delta^2 y_n - 7\Delta y_n + 12 y_n = 0$

Karena $\Delta = E - 1$, maka

$$(E-1)^2 y_n - 7(E-1) y_n + 12 y_n = 0$$

$$y_n \left[E^2 - 2E + 1 - 7E + 7 + 12 \right] = 0$$

$$y_n \left[E^2 - 9E + 20 \right] = 0$$

Substitusi $y_n = p^n$, maka :

PP : $p^2 - 9p + 20 = 0$

$(p-4)(p-5) = 0$

$p = 4 \vee p = 5$

Jadi, diperoleh : $y = C_1 4^n + C_2 5^n$ //

15). $y_{n+2} - 3y_{n+1} + 2y_n = 2^n$

PP : $p^2 - 3p + 2 = 0$

$(p-1)(p-2) = 0$

$p = 1 \vee p = 2$

PC : $U_c = C_1 + C_2 . 2^x$

IP : $U_p = \dfrac{1}{E^2 - 3E + 2} . 2^n$

$= \dfrac{2^n}{0}$ (Gagal)

Seharusnya : $U_p = \dfrac{1}{(E-1)} \dfrac{1}{(E\ 2)} . 2^n = \dfrac{1}{1} . \dfrac{1}{E-2} . 2^n$

$= 1 . 2^n . \dfrac{1}{2E-2}$

$= \dfrac{2^n}{2(1+\Delta)-2}$

$= \dfrac{2^n}{2\Delta}$

$= \dfrac{n . 2^n}{2}$

$= n . 2^{n-1}$

PUPL = $U_n = U_c + U_p$

$= C_1 + C_2 . 2^n + n . 2^{n-1}$

//

22). $(\Delta^2 + \Delta + 1) U_n = n^2$

$\left((E-1)^2 + (E-1) + 1\right) U_n = n^2$

$\left(E^2 - 2E + 1 + E - 1 + 1\right) U_n = n^2$

$\left(E^2 - E + 1\right) U_n = n^2$

PP : $p^2 - p + 1 = 0$

$p_1 = \frac{1}{2} + \frac{\sqrt{3}}{2} i$

$p_2 = \frac{1}{2} - \frac{\sqrt{3}}{2} i$

Pc = ambil $a = \frac{1}{2}$ $b = \frac{\sqrt{3}}{2}$

$r = \sqrt{a^2 + b^2} = 1$

$\left.\begin{array}{l} \sin\theta = \frac{b}{r} = \frac{\sqrt{3}}{2} \\ \cos\theta = \frac{a}{r} = \frac{1}{2} \end{array}\right\} \tan\theta = \frac{b}{a} = \sqrt{3}$

$\theta = \frac{\pi}{3}$

$U_c = r^n \left(C_1 \cos\frac{\pi}{3} n + C_2 \sin\frac{\pi}{3} n\right)$

$= C_1 \cos\frac{\pi}{3} n + C_2 \sin\frac{\pi}{3} n$

IP : $U_p = \dfrac{1}{E^2 - E + 1} n^2$

$= \dfrac{1}{(\Delta+1)^2 - (\Delta+1) + 1} n^2$

$= \dfrac{1}{\Delta^2 + \Delta + 1} n^2$

$= (1 - \Delta + \Delta^2 - \Delta^3 + \dots)(n^{(2)} + n^{(1)})$

$= (1 - \Delta)(n^{(2)} + n^{(1)})$

$= n^{(2)} + n^{(1)} - \Delta(n^{(2)} + n^{(1)})$

$= n^2 - (2n^{(1)} + 1)$

$= n^2 - 2n - 1$

Jawab : $U_n = U_p + U_c$

$= n^2 - 2n - 1 + C_1 \cos\frac{\pi}{3} n + C_2 \sin\frac{\pi}{3} n$

▶ Soal Latihan hal 66

4). $f_n = \{1, 2^2, 3^2, 4^2, 5^2, 6^2, \dots\}$

Jawab : $U_n = n^2$, $n = 1, 2, 3, 4, \dots$

$$Z\{n^2\} = \frac{z^2 + z}{(z-1)^3}$$

Dapatkan invers dari transformasi Z jika diberikan F(z) berikut :

4). $$F(z) = \frac{z^2 + z}{(z+1)(z+3)}$$

$$= \frac{z\cancel{(z+1)}}{\cancel{(z+1)}(z+3)}$$

$$= \frac{z}{z+3}$$

$$U_n = Z^{-1}\left[\frac{z}{z+3}\right]$$

$$= Z^{-1}\left[\frac{z}{z-(-3)}\right]$$

$$= (-3)^n$$